

Study of Faraday waves in tanks in presence of polystyrene bead layers

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Abstract. When a tank is subjected to vertical forced excitation, Faraday waves appear at the surface of the liquid the tank contains. In this paper, we consider the effect of layers of polystyrene beads placed on the surface of isopropanol undergoing a low frequency vertical sinusoidal excitation. Beads on the surface remove most of low-frequency resonances and reduce the amplitude of waves for the remaining ones. The formation of resonances with beads is observed to come from small gaps in the bead layers. A sufficient number of beads is needed to maintain beads in one compact block and prevent Faraday waves.

Keywords: Applied fluid mechanics / Resonance and damping of mechanical waves.

1 Introduction

Exciting a tank with forced excitation generates sloshing of the liquid it contains. Faraday waves are a special case of sloshing which appears when the tank is vertically excited [1–3]. This is illustrated in Figure 1. This phenomenon within launch vehicle tanks is a recurring problem when using liquid propellant engines. Many launchers, from the Soviet N-1 lunar rocket to the French Emerald series of launchers, have been lost due to sloshing [4]. Two phenomena are found. The first is called the “Pogo effect” [5–7]. This is a feedback phenomenon involving the propulsion, the structure and the liquids contained in the tanks of the launcher. During a flight, the sloshing of propellants in the tanks leads to pressure variations within the tanks. These pressure variations disrupt the power supply to the engine and cause variations in engine thrust. This causes vibrations in the launcher structures, which in turn causes the liquids in the tanks to slosh. When this phenomenon generates a positive feedback, it can lead to damage or destruction of the launcher. The second phenomenon is the application of lateral forces and torques due to sloshing. This can lead to problems with the trajectory [8] and can result in the destruction of the launcher. Mitigating propellant sloshing in launch vehicles is therefore a crucial challenge in the aerospace field but also in many fields of industry such as aeronautics, naval industry or liquid transport.

In order to limit sloshing, the various actors in the field favour structural modifications of the tank [9–11]. These modifications make the construction more complex and can increase the mass of the tanks. Other solutions exist, such as the presence of bubbles on the surface of the liquid [12,13]. However solving the issues involved by structural modifications, bubbles are complex to implement and maintain. The use of beads solves the constraints due to structural modifications and foams. In this paper, we consider the effect on Faraday waves of polystyrene beads placed on the surface of isopropanol in a rectangular tank subjected to low frequency vertical sinusoidal excitation. We present experimental visualisations of the surface motion with several bead layers and without. Four hypotheses will be considered to explain damping by polystyrene beads.

2 Method

2.1 Experimental setup

The experimental setup is shown in Figure 2. A rectangular tank of dimension $L = 70$ mm and $l = 15$ mm that is attached on a vibrating pot controlled with a function generator connected to an amplifier. The generator is set to the “sinusoid” mode, which creates a sinusoidal signal of selected amplitude and a shaking frequency f . The movement of the liquid was recorded with a Phantom Research v411 camera connected to a computer and set to record

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Fig. 1. Picture of isopropanol Faraday wave in a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled at $h = (32 \pm 2)$ mm.

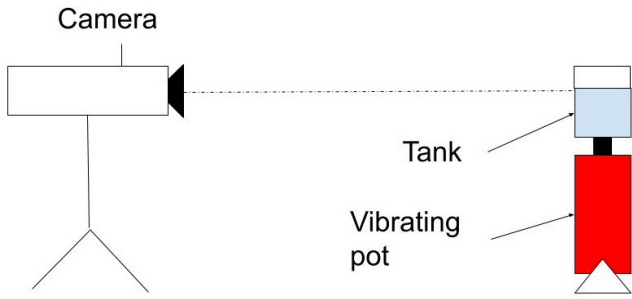


Fig. 2. Scheme illustrating the experimental setup [14].

300 fps in HD (1280×800). A LED lamp is used to illuminate the tank to ensure good video quality. The vertical oscillation amplitude of the pot is $A = (5 \pm 1)$ mm. The tank is filled with liquid to a height $h_{liq} = (32 \pm 2)$ mm. Due to the difficulty to excite eigenmodes in small tanks, isopropanol was selected because of its low surface tension. When polystyrene beads are involved in the experiments, they are placed at the surface of isopropanol such that they form a n -layer membrane. To measure the height of the content of the tank (either isopropanol or isopropanol + beads), the high-speed camera is placed 1.5 m away from the tank, far enough so that a proportionality relation between the distance measured in pixel and real vertical distances can be assumed. To calibrate the conversion from pixel to mm, a picture of a 20 cm ruler is used. Polystyrene balls with a diameter between 0.3 cm and 0.6 cm were used. The width of a bead layer corresponds to 3 polystyrene balls. Several measurements allowed us to evaluate the temperature of the isopropanol between 20 and 25 degrees, we will therefore consider that its density ρ is (782.9 ± 2.1) kg/m³ [15] and its surface tension γ is (21.48 ± 0.26) mN/m [16]. The gravity acceleration g is (9.81 ± 0.01) m/s² [17].

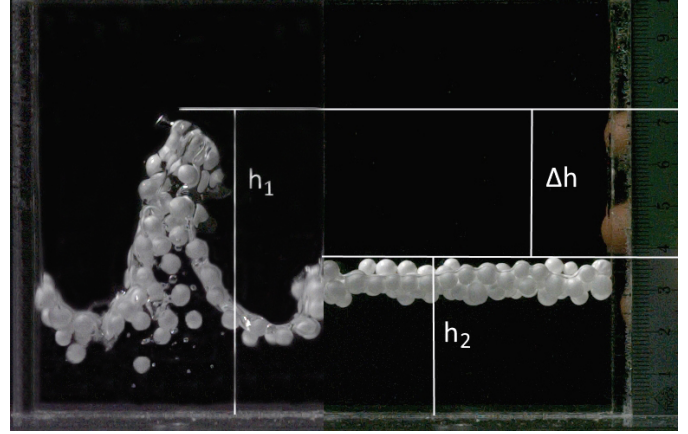


Fig. 3. Illustration of the measurement process. To obtain $\Delta h = h_1 - h_2$, a measure of the height of content at rest h_2 and the maximum height h_1 of the content of the tank under excitation is performed by using an image processing software.

2.2 Measure of maximal height difference

The wave amplitude is measured to characterise the impact of beads on Faraday waves' damping. A protocol to measure this amplitude is presented below. A movie is made with the camera once the steady state is reached. The maximum height h_1 of the content of the tank is then obtained and measured with the image processing software "paint.net" by counting the number of vertical pixels between the bottom of the tank and the highest point of the content. To obtain $\Delta h = h_1 - h_2$, a measure of the height of content at rest h_2 is performed with and without beads. This measurement process is illustrated in Figure 3.

2.3 Swap frequency with and without beads

In order to see damping by beads at low frequencies, a shaking frequency swap is performed between 0 and 10 Hz every 0.2 Hz with and without two layers of polystyrene beads. In each case, a measurement of the maximal height difference Δh is made.

2.4 Impact of the number of bead layers

To assess the influence of the number of bead layers, a measurement is performed with number of layers between 0 and 4 for a shaking frequency f of 9, 9.6 and 10 Hz. In each case, a measurement of the maximal height difference Δh is made.

3 Results

3.1 Swap frequency with and without beads

The experimental results for swap frequency are presented in Figure 4. Introducing a layer of beads makes it possible

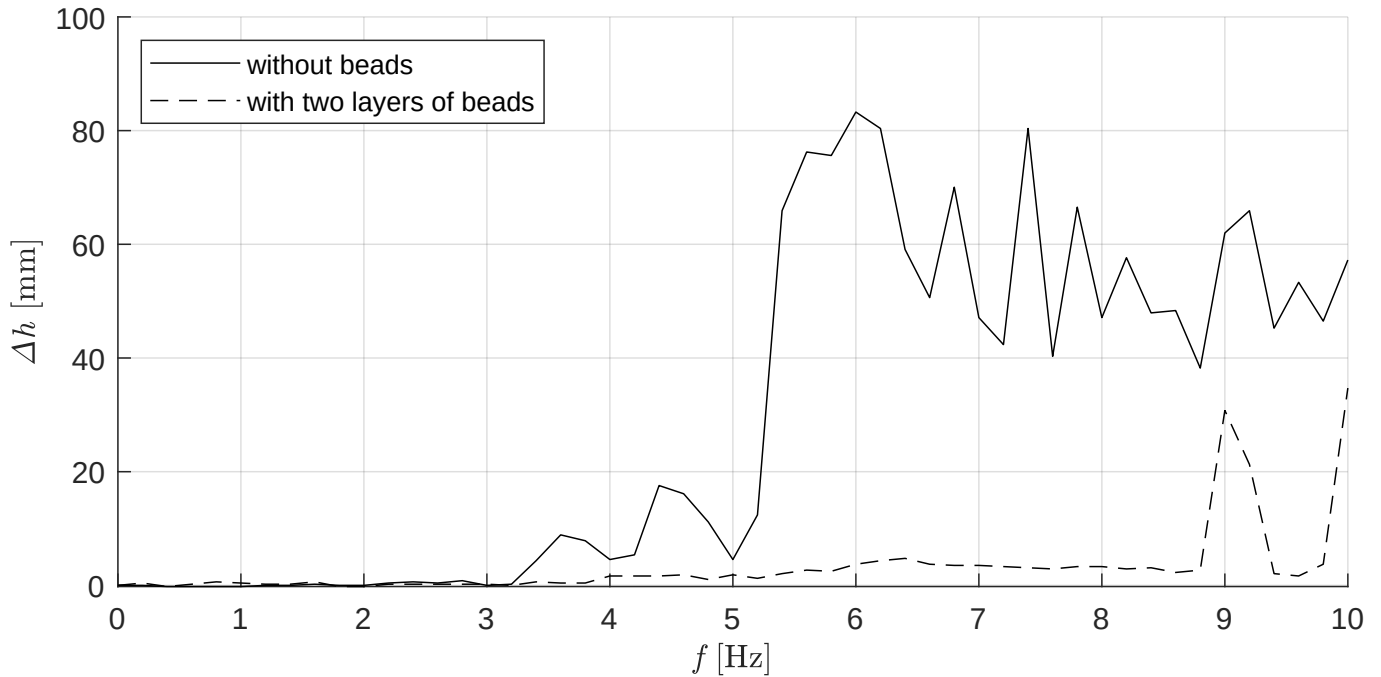


Fig. 4. Maximum height difference Δh of the liquid's surface with and without two layers of beads for a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled at rest at $h_{\text{liq}} = (32 \pm 2)$ mm for shaking frequencies f between 0 Hz and 10 Hz.

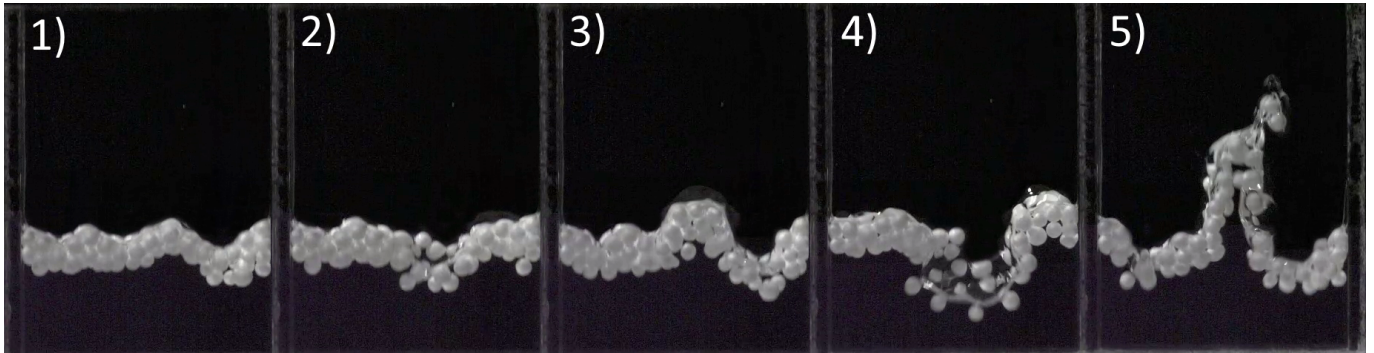


Fig. 5. Photo series of isopropanol Faraday wave with two layers of beads at (9.000 ± 0.005) Hz in a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled with liquid at rest at $h_{\text{liq}} = (32 \pm 2)$ mm which illustrates the phenomenon of continuity break of the bead block.

to significantly reduce the oscillations without however eliminating them completely (Fig. 4). Slight oscillations do remain. Around a shaking frequencies of 9 and 10 Hz, it was observed that there were still significant oscillations, although less than without beads. This phenomenon is illustrated in Figure 5. It is observed that for these frequencies, the surface oscillations are too important to keep a continuous layer of beads on the surface (Images 1 and 2). A strong excitation (Images 3 and 4) appears in the area not covered entirely by beads and spreads to the rest of the surface (Image 5).

3.2 Impact of the number of bead layers

The impact of the number of bead layers is presented in Figures 6, 7 and 8, respectively for shaking frequencies

$f = 9$ Hz, $f = 9.6$ Hz and $f = 10$ Hz. It shows that Δh decreases with an increasing number of beads.

4 Discussion

4.1 Faraday waves

The liquid oscillates in the subharmonic regime, i.e. its oscillation frequency corresponds to half the shaking frequency of the vibrating pot. This is consistent with the characteristics of Faraday waves [2]. For small oscillations up to $\Delta h = 20$ mm, the waves are observed to be periodically oscillating sines. For larger oscillations, the regime is quasi-periodic. The shape of the surface is chaotic, but the period and the measured Δh are constant once the quasi-periodic regime is reached.

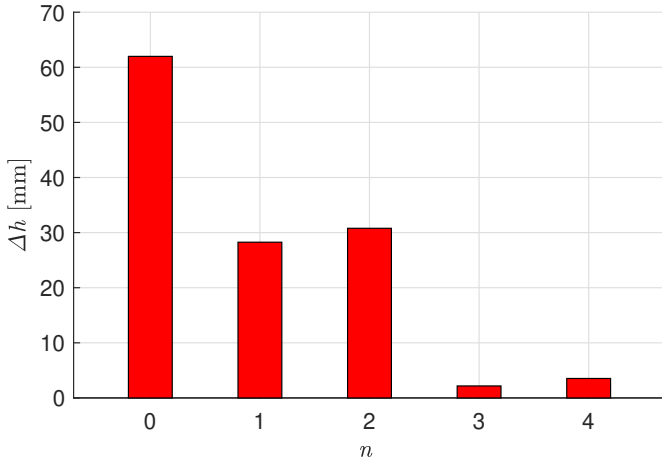


Fig. 6. Maximum height difference Δh for a number of bead layers n between 0 and 4, at 9 Hz in a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled with liquid at rest at $h_{\text{liq}} = (32 \pm 2)$ mm.

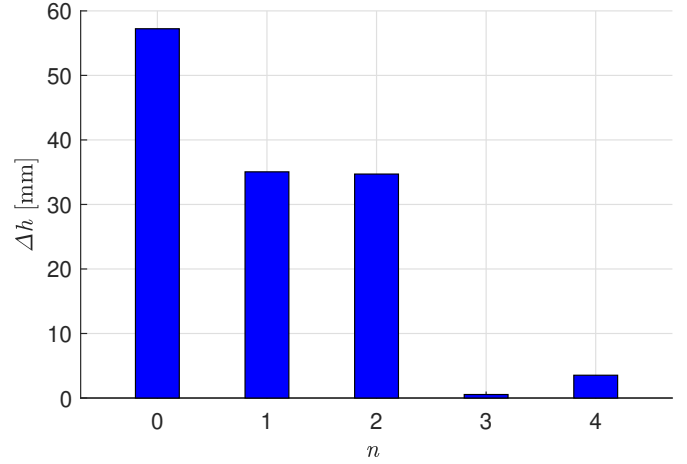


Fig. 8. Maximum height difference Δh for a number of bead layers n between 0 and 4, at 10 Hz in a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled with liquid at rest at $h_{\text{liq}} = (32 \pm 2)$ mm.

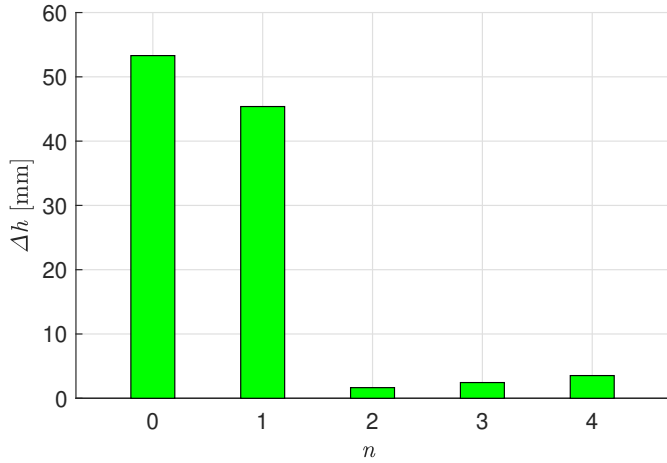


Fig. 7. Maximum height difference Δh for a number of bead layers n between 0 and 4, at 9.6 Hz in a rectangular tank of dimensions $L = 70$ mm and $l = 15$ mm filled with liquid at rest at $h_{\text{liq}} = (32 \pm 2)$ mm.

4.2 Swap frequency with and without beads

Figure 4 shows that beads have a significant impact on damping Faraday waves. By comparison with damping by foam on the surface [12], one can suggest that the main contribution to damping comes from the beads in contact with the walls. The low density of the beads allows them to aggregate on the surface and thus maximise the static friction between them, so that the beads form a compact block which is held together and immobile relative to the tank. This compactness strengthens the friction with the wall and thus increases energy dissipation.

Beads constitute small items at the surface of the liquid, therefore they are obstacles that can break waves and inhibit the formation of new waves. They are very effective in preventing resonance modes from being excited (see Fig. 4). These beads can also be considered as an

elastic membrane [18]. This membrane applies a pressure on the liquid's surface, restraining its movement. The Faraday waves' amplitude is reduced and resonances are shifted to higher frequencies. Therefore, this shift could be useful in the aerospace field as vibrations of rockets' structures are mainly low in frequencies [4]. The same phenomena are observed if the membrane is replaced by a thin layer of a viscous and lower density liquid on the surface. Treating beads this way makes it easier to compute numerically the damping by solving with the Navier–Stokes equation [3]. In this case the beads' effective surface tension and effective viscosity need to be measured experimentally.

4.3 Impact of the number of bead layers

Figures 6, 7 and 8 show that Δh decreases with an increasing number of beads. For 3 and 4 layers, the observed Δh is not attributed to Faraday waves but instead to a few beads of the last layer which jump over the compact bead block. This is an artefact of the measurement process, the wave is not better damped with 3 layers than with 4 as in both cases there is no wave. The same intensity is observed with 1 and 2 layers. Figure 4 shows that 9 and 10 Hz correspond to resonances in the 2-layer scenario, which may not be the case with 1 layer as beads shift the resonances. This hypothesis is confirmed by the experiments at 9.6 Hz, as there are waves with one layer but not with two. To guarantee a maximal damping of resonances by beads, the surface continuity of beads needs to be insured. Increasing the number of layers of beads prevents the formation of gaps in the bead membrane as they are immediately filled with beads from an upper layer, therefore preventing discontinuities of the bead membrane. The friction with the wall is also proportional to the number of beads in contact with these walls, therefore it is also proportional to the number of layers of beads. For a tank excited by shaking the tank horizontally, it has been shown in literature

that the number of layers is positively correlated with damping [19].

5 Dead end

5.1 Experimental setup

At first, we did our experiments with water, however, even with a single layer of beads no wave could be observed for any frequency nor shaking amplitude (considering the limits of the vibrating pot). The only conclusion was that beads were efficient, but the process of how they inhibit Faraday waves does not allow any quantitative analysis. Isopropanol is a liquid with a smaller surface tension therefore easier to excite. As its viscosity is lower, damping of waves is reduced so that the impact of beads is clearer. It is also cheap and easy to acquire. Initially, we planned using a cylindrical tank for our experiments since the tanks of the aerospace industry where the use of balls would be usable are of this shape. However, we found that these tanks are not suitable for our measurement protocol due to its width. Indeed, the cylindrical tanks have a width equal to their diameter. The upper surface liquid profile cannot be projected in 2D from the camera point of view. Therefore the maximum height cannot be observed visually by only one camera (see Fig. 9). However, equations (1) and (2) [2], which correspond to the resonance frequencies of a perfect fluid respectively in a rectangular tank and in a cylindrical one, are very similar, the only difference being the geometrical coefficient k_{mn} (resp. λ_{mn}). As the physics of these tanks is similar, we carried out our studies on a rectangular one, considering that the results thus obtained can be applied to cylindrical ones.

$$f_{mn} = \frac{1}{2\pi} \sqrt{\left[g k_{mn} + \frac{\gamma}{\rho} k_{mn}^3 \right] \tanh(k_{mn} h)} \quad (1)$$

where $k_{mn} = \pi \sqrt{m^2/L^2 + n^2/l^2}$, l , L and h are respectively the width, length and height of the rectangular tank, g is the gravitational acceleration, γ is the surface tension of the liquid and ρ is its density.

$$f_{mn} = \frac{1}{2\pi} \sqrt{\left[g \lambda_{mn} + \frac{\gamma}{\rho} \lambda_{mn}^3 \right] \tanh(\lambda_{mn} h)} \quad (2)$$

where $\lambda_{mn} = \varepsilon_{mn}/R$, R is the radius of a cylindrical tank, ε_{mn} is the n th root of the derivative of J_m , J_m is the first kind Bessel function of order m , g is the gravitational acceleration, γ is the surface tension of the liquid and ρ is its density. Using equation (1) with the setup used for Figure 4, the three lowest resonant frequencies without beads are found to be 3.17 Hz, 4.76 Hz and 5.93 Hz. However the first resonances in Figure 4 are at (3.7 ± 0.2) Hz, (4.4 ± 0.2) Hz (6 ± 0.5) Hz. The right order of magnitude is predicted, but resonances are shifted to lower frequencies due to the strong non-linearity of the oscillation regime for large amplitudes.

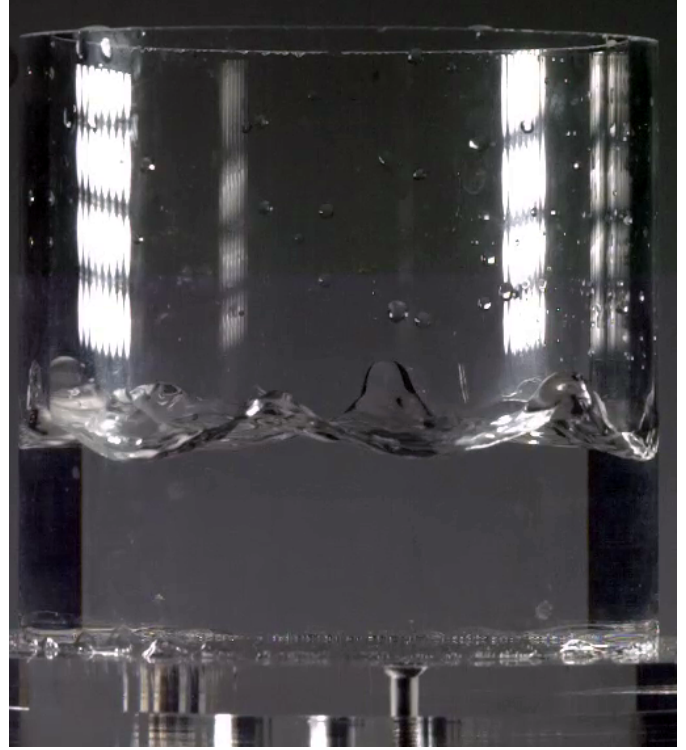


Fig. 9. Picture of Faraday waves in a cylindrical tank of dimensions $R = 5$ cm and filled with liquid at rest at $h_{\text{liq}} = (4 \pm 2)$ mm.

5.2 Swap frequency with and without beads

We chose the main frequency swap presented in Figure 4 with two layers for the case with beads. We tried with many more bead layers at first but no sloshing could be observed. With two layers we estimated that the effect of beads could be observed without inhibiting the oscillation, so that resonance frequency shifts and damping could be analysed.

5.3 Impact of number of bead layers

We chose to study the impact of the number of layers at frequencies where sloshing was significant. For two layers, Figure 4 shows that between 9 and 10 Hz, there is sloshing even with two layers of beads. Between 4 and 5 beads fit widthwise between the walls of the tank. No transverse bridges were observed in any of the experiments.

6 Conclusion

In this paper, the influence of polystyrene beads on the surface of a liquid under forced excitation is studied in order to attenuate Faraday waves. It is concluded that the presence of layers of beads on the surface of a liquid allows to damp the oscillations. The formation of resonances with beads is observed to derive from little gaps in the bead layers, where the liquid can start to oscillate. Once this small oscillation is triggered, it propagates to the whole tank in

a short time. A sufficient number of beads is needed to maintain beads in one compact block and prevent discontinuities of the bead layers. Four hypotheses are advanced to explain this phenomenon: friction of beads with the tank's walls contribute to damping ; beads mechanically break the waves; beads can be considered as a thin elastic membrane restraining the motion of the liquid; beads can be considered as a liquid, with an effective surface tension and an effective viscosity.

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