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# Modelling of energy transition as an optimization problem: is it possible to achieve energy transition in France without renewable energy research?

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**Abstract.** To illustrate the energy transition concept and its associated stakes, we used the mathematical optimization formalism and designed a *toy-model* applied to the French energy system. This approach is not predictive but lays on four pedagogical scenarios considering industrial capacity and resources limitation, and recycling process to characterise a field of possibilities. Thus, productivity limit and lifetime of solar panels emerge as crucial factors in the short term. Massive recycling process seems to delay material scarcity effect to long term. In contrast, increasing demand remains a significant burden to energy production sustainability.

**Keywords:** Energy transition / mathematical optimization / toy model / photovoltaic / planetary boundaries.

## 1 Introduction

To present the issues related to energy transitions, we designed a simple modular model of the French energy system, implemented as a linear optimisation problem. It is not a predictive model, but a *toy model* to experiment with effects of assumptions on the success of a transition. It aims at outlining the field of possibilities. We explicitly set aside performances improvement, i.e. renewable energy research outcomes, to assess the practicability of a transition with current mature technology. To do so, mathematical optimization (MO) programming enable us to determine the feasibility of a sustainable energy transition based on an objective and given constraints [1], including limitation of industrial capacity, existence of recycling process and energy needs. Optimization, in our case, results in computing optimal energy mixes and material consumption over time.

## 2 Method

Tools and problem definition.

### 2.1 Numerical tools

We used *AMPL IDE* version *3.6.1* with solver *CPLEX 12.10.0.0* and created figures with *Excel 2016* and *Python* version *3.8.2*.

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### 2.2 Problem definition

We consider a factory that can produce a number $\mathbf{m}(\mathbf{t})$ of photovoltaic panels (*noted PV panels*) per year. Then, each variable is discrete and depends on time $\mathbf{t}$. The year 2010 is considered as the time origin of the runs, which is justified in the *Grid demand D(t)* section. $\mathbf{N}(\mathbf{t})$ is the number of operational solar panels and $\mathbf{N_{ooo}}(\mathbf{t})$ the number of solar panels out of order after an operational period, at a time t. Initially we consider both as empty stocks, to simulate an energy transition from scratch. $\mathbf{Q_y}(\mathbf{t})$ and $\mathbf{Q_x}(\mathbf{t})$ represents the stocks of fossil energy and matter as discrete functions of time. The material consumption of the factory $\mathbf{xc_f}(\mathbf{t})$ is sustained by mining production $\mathbf{xp_e}(\mathbf{t})$.

The energy production $\mathbf{yp_{PV}}(\mathbf{t})$ resulting from the $\mathbf{N}(\mathbf{t})$ solar panels must meet the energy demand of the grid $\mathbf{D}(\mathbf{t})$, to which energy consumption of the extraction production $\mathbf{yc_e}(\mathbf{t})$ and the PV panels factory $\mathbf{yc_f}(\mathbf{t})$ is added. If photovoltaic energy production does not meet the demand, fossil energy $\mathbf{yp_{ff}}(\mathbf{t})$ completes it. In parallel of the PV factory, and to account for end-life treatment issues, a recycling factory with a productivity $\mathbf{m_r}(\mathbf{t})$ converts a part of the panels out of order into recycled raw material (expressed by the variable $\mathbf{xp_r}(\mathbf{t})$ in kg per year) in exchange of a supplementary energy consumption $\mathbf{yc_r}(\mathbf{t})$. The whole process is visible in Figure 1, where stocks and energy mix are illustrated with squares and flows, associated to variables, with arrows. Table 1 sums up the variables of the problem, by PV panel numbers variables (unitless), energy variables expressed in kilo

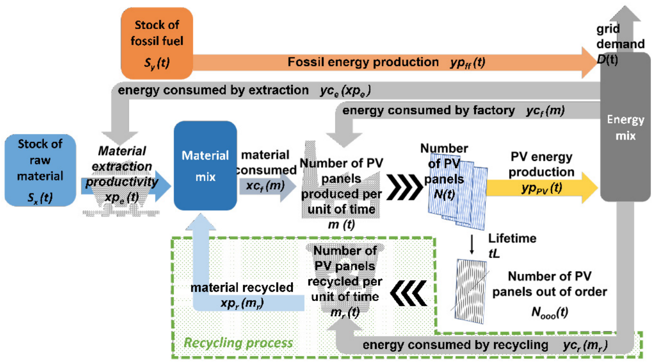


**Fig. 1.** Modular diagram representing interactions between PV panels factory, matter and energy stocks, and energy mix. The recycling module is surrounded in a green frame, its implementation depends on the chosen scenario to run. Same for grid energy demand $D(t)$.

**Table 1.** Discrete variables of the model depending on time $t \in [\![0, t_f]\!]$, with $\mathbf{t_f}$ a positive integer as the time horizon of a run.

| PV panel number variables | $(\in \mathbb{N}^{\mathbf{t_f}})$ |
|---|---|
| $m(t)$ | yearly productivity of the PV panels factory |
| $m_r(t)$ | yearly productivity of the recycling factory |
| $N(t)$ | number of PV panels |
| $Nooo(t)$ | number of broken panels |
| **Energy variables** | $(\in (\mathbb{R}^+)^{\mathbf{t_f}})$ |
| $yp_{PV}(t)$ | yearly PV energy production |
| $yp_{ff}(t)$ | yearly fossil fuel production |
| $yc_f(t)$ | yearly energy consumption of the PV panels factory |
| $yc_e(t)$ | yearly energy consumption of mining extraction |
| $yc_r(t)$ | yearly energy consumption of the recycling factory |
| $S_y(t)$ | stock of fossil energy |
| $D(t)$ | energy demand of the grid |
| **Matter variables** | $(\in (\mathbb{R}^+)^{\mathbf{t_f}})$ |
| $xc_f(t)$ | yearly material consumption |
| $xp_e(t)$ | yearly extraction of raw material |
| $xp_r(t)$ | yearly material recycling |
| $S_x(t)$ | stock of raw material |

Watt hour and matter variables in kilogram. Green lines present variables related to the recycling factory.

Each of theses variables will have its evolution built over the run to optimise an energy transition based on a chosen scenario. Moreover, we choose to apply this model to France, which implies for instance to estimate French photovoltaic potential and French electricity demand.

### 2.3 Parameters of the model

The evolution of model's variables depends on characteristics of the industry (productivity, raw material availability) and on the photovoltaic technology (efficiency, durability). Consequently, we set the six following assumptions on the characteristics of a standard solar panel based on the literature, given an order of magnitude perspective.

**Hypothesis of the model:**

1) A panel is monomaterial and monocristalin, made of silicon;
2) The surface of a panel is $1.7\,\mathrm{m}^2$, its thickness is $200\,\mu\mathrm{m}$ and weighs 0.8 kg [2];
3) A panel presents in average 0.3 kWp of capacity [3];
4) A panel operates for 25 years [4];
5) The energy-capacity ratio of a PV panel is constant over its life-time and is the same as in the South of France; which is equal to 1.3 kWh/kWc [5];
6) The yearly sunshine hours is 2000 h in South of France [6];
7) "We need about 6 tons of raw material to produce 1 ton of silicon" [7];

Then, recycling process implies the two following hypothesis:

8) Panel's material is recycled up to 95% [8];
9) The energy savings of a panel recycling is 15% [9]. This is an optimistic hypothesis according to silicon properties.

Lastly, 10) photovoltaic energy conversion does not emit $CO_2$, and fossil energy consumption has a fixed conversion coefficient of 0.65 kg of equivalent $CO_2$ per kWh, which corresponds to use oil combustion to generate power [10]. Concerning PV $CO_2$ emissions, it is a strong assumption which allows to assimilate the energy transition objective of a run to a zero emissions objective.

Based on these assumptions, we computed three additional parameters:

- $yp_{PV}^{(1)}$, the yearly conversion of PV energy per panel

$$yp_{PV}^{(1)} = (energy\_capacity\ ratio) \cdot (panel's\ capacity) \cdot (sunshine\ hours) \quad (1)$$

- $yc_f^{(1)}$: the energy consumption to manufacture one PV panel. It includes the following steps: crystallization and contouring, Czochralsky process, wafering, cell processing module assembly, frame, add of electronic devices and human labour . The maximum values of frameworks are chosen but corrected by a factor 0.8/2 because module weight is 2 kg in references [11] and [12]), but only 0.8 kg in our case (see hypothesis 2) ).
- $yc_e^{(1)}$: the energy consumption to extract the raw material quantity equivalent to one PV panel. It includes extracting [13] and purification [12].

Note that mining and manufacturing account respectively for 33% and 66% of the overall embodied energy of a PV panel. Moreover, the Energy Returned On Energy Invested (ERoEI) is equal to 5.95. More broadly, the set

**Table 2.** Technological parameters of the model. "/pnl" means "per panel".

| Parameter | Value | Unit | Description |
|---|---|---|---|
| $yp_{PV}^{(1)}$ | $7.8 \times 10^5$ | Wh/pnl /year | PV energy conversion |
| $yc_f^{(1)}$ | $2.17 \times 10^6$ | Wh/pnl | factory's energy consumption |
| $yc_e^{(1)}$ | $1.38 \times 10^6$ | Wh/kg | extracting energy consumption |
| $yc_r^{(1)}$ | $1.18 \times 10^6$ | Wh/pnl | recycling energy consumption |
| $xc^{(1)}$ | 0.8 | kg/pnl | factory's material consumption |
| $xp_r^{(1)}$ | 0.76 | kg/pnl | recycling material production |
| $C_{CO_2}$ | $6.5 \times 10^{-4}$ | kg/Wh | conversion coefficient of $CO_2$ emission |
| $t_L$ | 25 | year | panel life time |

of parameters (see synthesis Tab. 2) and by extension the model's results hinge on the assumption of constant characteristics, i.e. the absence of improvement through research, all along the simulation. To account for the industrial component of the problem, flows and stocks hypotheses are added to the model:

11) Factory's productivity is limited;
12) Fossil energy stock and raw material stock are finite;
13) Recycling intensity, when it is implemented, is limited only by the existing quantity of PV panel out of order;
14) Energy demand of the grid can increase due to the population growth.

These hypotheses will be discussed in the result section with four scenarios highlighting specific aspects.

To make hypothesis 11) and 12) explicit, we define a maximum yearly productivity $M_{\max}$, finite initial stocks of fossil energy and raw material, respectively $Qy_{init}$ and $Qx_{init}$. Likewise, the multiplicative coefficient $\alpha$ imposes a linear increasing trend on the energy demand of the grid.

- $M_{\max}$: It corresponds to the limitation of the productivity of the PV panels factory, assumed proportional to the maximum photovoltaic capacity installed between 2009 and 2019 [14]. Considering a standard PV panel with fixed performances (cf. Hypothesis 3), it leads to the following equation:

$$M_{max} = \frac{\text{Capacity installed in 2011}}{\text{Capacity of 1 panel}} \quad (2)$$

  This parameter embodies manufacturing and installation of panels. This calculation gives order of magnitude of the current industrial limits. We will apply this limitation to all the run time to discuss energy transition phenomenon.

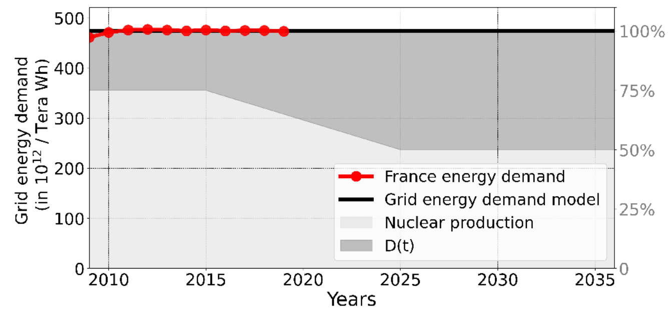


**Fig. 2.** Representation of the energy demand of France (red dot), the associated linear model (gray line), the scenario of the nuclear French production according to the "energy transition law" of 2015 (area in light gray) and the resulting low-carbon energy demand $D(t)$ (area in gray).

- $Qx_{init}$ and $Qy_{init}$ have no realistic order of magnitude. They will be defined relatively to the model behaviour to experiment scarcity effect or carbon budget.
- $\alpha$: increasing demand factor of the energy demand. It varies between 0 (constant demand) and 0.1% per year for the most pessimistic scenario [15].

In addition to these three parameters, fossil fuel consumption has to be compared with the quantity $Qy_X$ °C, based on $B_{F_X}$ °C, the $CO_2$ global budget to mitigate global warming below $X$ °C. For instance, global budget $B_{F_2}$ °C was about 1010 Gt in 2011 [16], and has to be distributed to countries and industrial sectors according to an allocating strategy. In our study we selected a demographic strategy, attributing to France a $CO_2$ equivalent budget about 0.9% of the global budget, because France population account for 0.9% of the global population in 2011 [17]. Moreover, power production accounts for 41% of total $CO_2$ emission [18]. Thus:

$$Qy_{2\,^\circ\mathrm{C}} = \%_{Power_{\mathrm{prod}}} \%_{POP_{\mathrm{France}}} \times \frac{B_{F_{2\,^\circ\mathrm{C}}}(2011)}{C_{\mathrm{CO_2}}} = 41\% \times 0.9\% \frac{1010\,\mathrm{Gt}}{0.65\ \mathrm{kg/kWh}} = 5.73 \times 10^{15}\ \mathrm{Wh}. \quad (3)$$

Similarly, for a 1.5°$C$ budget:

$$Qy_{1.5\,^\circ\mathrm{C}} = 41\% \times 0.9\% \frac{540\,\mathrm{Gt}}{0.65\,\mathrm{kg/kWh}} = 3.07 \times 10^{15}\ \mathrm{Wh}. \quad (4)$$

### 2.4 Grid demand $D(t)$

Grid energy demand is modelized based on two assumptions: France energy demand linearly depends on time (considered constant in a first time according to demand data [19], see Fig. 2). Moreover, French nuclear production follows the energy mix imposed by the French Energy Transition law which intends to reduce nuclear component from 75% to 50% before 2025 [20]. Thus, the complement of nuclear production is considered as low-carbon energy demand associated to $\mathbf{D(t)}$.

**Table 3.** Industrial parameters of the model.

| Parameter | Value | Unit | Description |
|---|---|---|---|
| $M_{\max}$ | $10^7–1.8 \times 10^8$ | panels per year | maximum productivity |
| $Qy_{init}$ | $10^{16}$ | Wh | fossil energy initial stock |
| $Qx_{init}$ | $10^9–10^{10}$ | kg | raw material initial stock |
| $\alpha$ | 0–0.1 | % | Increasing demand factor |

Figure 2 presents then a transitory regime (before 2025), corresponding to the "Energy Transition law" and an permanent regime (after 2025) where the part of nuclear production in energy mix is assumed constant and equal to 50%. This construction of the demand, added to the fact that the maximum installed PV capacity since 2010 was in 2011, imposes 2010 as the time origin of our model. Table 3 synthesizes the industrial parameters and range values we will consider in our scenarios.

We will investigate each parameter's effect on the result using several scenarios. We now introduce the mathematical optimization model, based on an objective function and 16 mathematical constraints.

## 2.5 Objective function

Considering the problem, we define a "good energy transition" objective, which means to reach rapidly and definitely an energy mix of 100% renewable. It is equivalent to minimize the cumulative fossil energy consumption over all the duration of the simulation, and make the yearly consumption decreasing over the simulation (a). Thus, the objective function will be the following one for all the scenarios:

$$\text{minimize} \sum_{t=1}^{t_f} yp_{ff}(t). \tag{5}$$

where $t_f$ is considered the time horizon. Results have to be compared with the fossil energy budgets $Qy_{X\,^\circ\mathrm{C}}$.

## 2.6 Constraints

The black constraints ((a) to (g) and (k) to (p)) are always operational, when the green constraints ((h) to (i)) are activated only when recycling process is effective (i.e. in scenario C and D). Some of the constraints are also applied before or from $tl$, the first panels lifespan.

- Constraint (a) Fossil energy consumption has to decrease during all the run;
- Constraint (b) Annual production of PV panels is limited by a constant parameter $M_{max}$;
- Constraint (c) Manufacturing consumes energy and material, proportionally to the productivity;
- Constraint (d) Extracting consumes energy proportionally to extracted raw material quantity;
- Constraint (e) Fossil energy feeds the total energy demand subtracted of the renewable energy production;
- Constraint (f) Raw material extracting feeds the material demand subtracted of the matter recycling production;
- Constraint (g) Fossil energy stock and raw material stock are depleted according to extraction and fossil fuel consumption;
- Constraint (h) Recycling consumes energy and produces material, proportionally to its productivity;
- Constraint (i) Annual recycling productivity is limited by the number of recyclable panels.

These constraints are implemented as follows:

$$\forall t \in [\![1, tf-1]\!], \qquad yp_{ff}(t+1) \leq yp_{ff}(t) \tag{a}$$

$$\forall t \in [\![1, tf]\!], \qquad 0 \leq m(t) \leq M_{\max} \tag{b}$$

$$\begin{cases} yc_f(t) = m(t) \cdot yc_f^{(1)} \\ xc_f(t) = m(t) \cdot xc_f^{(1)} \end{cases} \tag{c}$$

$$yc_e(t) = xp_e(t) \cdot yc_e^{(1)} \tag{d}$$

$$yp_{ff}(t) = \overbrace{D(t) + yc_f(t) + yc_e(t) + yc_r(t)}^{\text{Total energy demand}} - yp_{PV}(t) \tag{e}$$

$$xp_e(t) = xc_f(t) - xp_r(t) \tag{f}$$

$$\begin{cases} Sy(t) = Sy(t-1) - yp_{PV}(t) \\ Sx(t) = Sx(t-1) - 6 \cdot xp_e(t) \end{cases} \tag{g}$$

$$\begin{cases} yc_r(t) = m_r(t) \cdot yc_r^{(1)} \\ xp_r(t) = m_r(t) \cdot xp_r^{(1)} \end{cases} \tag{h}$$

$$m_r(t) \leq Nooo(t) \tag{i}$$

Initial conditions are imposed by *Constraint (j)* Stocks are initially full, and *Constraint (k)* number of panels (working or not) is 0, both equivalent to:

$$\begin{cases} Sy(0) = Qy_{init} \\ Sx(0) = Qx_{init} \end{cases} \quad \text{(j)} \qquad N(0) = 0 \quad \text{(k)}$$

Furthermore, the number of panels has to be computed depending on time and relatively to the panel life time

$t_L$: When $t < t_L$, number of functional panels increases proportionally to productivity (*Constraint* ($l$)) and there is no panels out of order (*Constraint* ($m$)). Thus, $\forall t \in [\![1, tl]\!]$,

$$N(t) = N(t-1) + m(t) \quad \text{(l)} \qquad Nooo(t) = 0 \quad \text{(m)}$$

When $t > t_L$, number of functional panels depends also on the out of order panels number (*Constraint* ($n$)) and number of panels out of order increases (*Constraint* ($o$)).

$$\forall t \in ]\!]tl, tf]\!], \quad N(t) = N(t-1) + m(t) - m(t - tl) \quad \text{(n)}$$

$$Nooo(t) = Nooo(t-1) + m(t - tl) - m_r(t) \quad \text{(o)}$$

Finally, to mitigate the boundary effect of the optimization, we impose an increase of the PV energy production for the five latest years of the run (*Constraint* $p$)).

$$\forall t \in [\![tf - 5, tf]\!], \qquad yp_{PV}(t) \leq yp_{PV}(t+1) \quad \text{(p)}$$

## 3 Results

Four scenarios have been studied.

### 3.1 Scenario A: Influence of factory's productivity limit

Without recycling and with infinite raw material stock. Figure 3 presents the energy mix resulting from scenario A, with $M_{\max}$ equal to 10 million panels per year (up) and equal to 15 million per year (down) and with $t_L = 100$ years. Both seem to follow the same pattern: a first period of linear increase of the PV production and when the life time of the first produced panels is overtaken, PV production reaches a plateau. However, a productivity of ten million panels per year is too limiting to ever reach a free carbon energy mix.

Parameter $M_{\max}$ is proportional to the slope of the PV production in the increasing period. Thus, it strongly affects the success of the energy transition. To synthesize the effect of $M_{\max}$ on the success of the energy transition, Figure 4 presents productivity limitation on x-axis and total fossil fuel consumption over the simulation on $y$-axis, equal to the orange surface on Figure 3. Each point corresponds to a run with $t_f = 100$ years. Thus, we find the runs of Figure 3 on $x$-axis respectively at 10 million per year (in gray, meaning the failure of the energy transition) and at 15 million per year (in black, meaning that final energy mix is 100% renewable).

When $M_{\max} = 18 \times 10^7$ pnls/year, an optimal strategy which minimizes the total fossil fuel consumption is to build huge quantity of panels the first year, reaching a 100% renewable energy mix in only one year.

Moreover, our model makes structurally achieving an energy transition impossible after exceeding the panel life time (25 years). Finally, it appears that when energy transition is achieved, resulting total fossil fuel consumption remains below the lower carbon budget. For the next scenarios, we will attribute to $M_{\max}$ the value of 15 million panels per year.

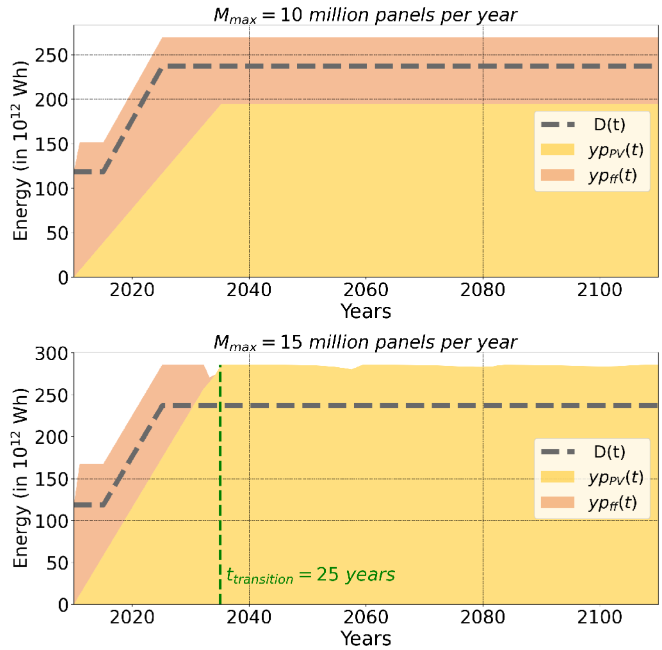


**Fig. 3.** Energy mix of France following the scenario A over 100 years, varying limitation of factory productivity $M_{\max}$, from $1 \times 10^7$ (upper) to $1.5 \times 10^7$ (lower) panels per year. Fossil fuel production (in orange), PV production (in yellow) and grid energy demand (dashed black line). Transition date $t_{\text{transition}}$ (in green) pinpoints when a 100% PV energy mix is reached.

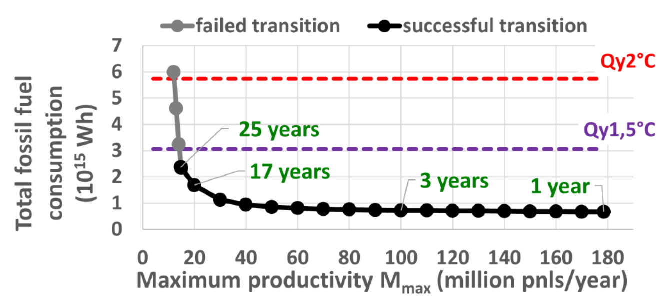


**Fig. 4.** Total fossil fuel consumption of runs with varying maximum productivity $M_{\max}$, with $t_f = 100$ years. Each dot corresponds to a run with a specific $M_{max}$. Gray and black dots correspond respectively to failed and successful energy transition along with its duration (in green). Dashed lines represent power generation carbon budget of France to maintain global warming below 1.5 °C (in purple) and 2 °C (in red).

### 3.2 Scenario B: Raw material scarcity

This scenario aims at examining the effect of the limitation of raw material on the energy transition. To do so, we compute runs with initial stock of raw material $S_x(0)$ lower than what was consumed in scenario A, that is $S_x(0) = 6 \times 10^6$ kg.

Figure 5 shows the resulting optimized energy mix over 100 years. This figure illustrates a scarcity effect, implying

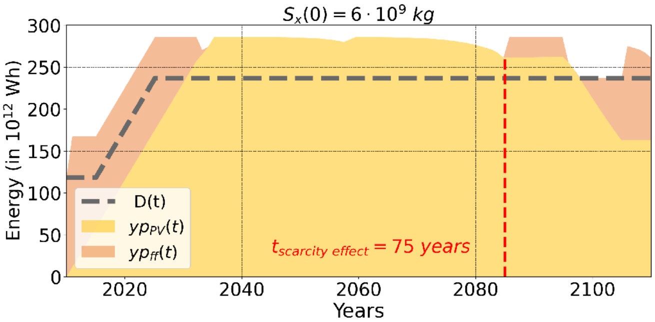


**Fig. 5.** Energy mix of France following the scenario B with $S_x(0) = 6$ billion kilograms and $t_f = 100$ years. Highlighting the scarcity effect on the PV production after 75 years.

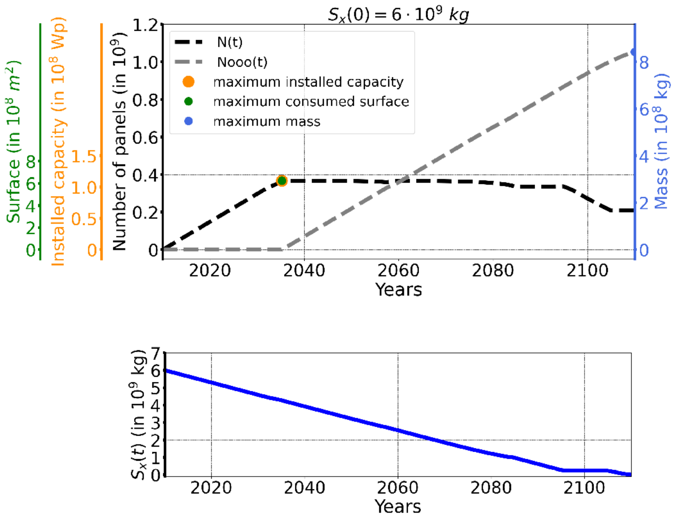


**Fig. 6.** Operational panels (black dashed line) and out of order panels (gray dashed line) (up), and stock of raw material (blue line) as functions of time, following the scenario B with $S_x(0) = 6$ billion kilograms. and $t_f = 100$ years.

the re-increase of fossil fuel consumption after a period of full free carbon energy mix, due to the lack of material to produce new PV panels. In other terms, the determined solution can no longer satisfy constraint (a) after time $t_{\text{scarcity effect}}$. The bottom part of Figure 6 shows that PV production requires a continuous consumption of material which in linear until 2090. It could be considered as a energy/material conversion phenomena.

Remark that this trend should deplete the initial stock $S_x(0)$ at the horizon of 2100, leading to the collapse of PV panels number, then of the PV production and as a consequence the re-increase of fossil fuel consumption. However, instead of consuming the entire raw material stock around 2100, a plateau appears before finally declining until zero. It is a model default which account for the duration of the run in the optimization that we called *boundary effect*. Thus, scarcity leads to failure of the energy transition.

Figure 7, based on the structure of Figure 4, plots points as synthesis of energy mix runs while dissociating which one reaches a final full free carbon energy mix (black dots) and others (gray dots), considering varying initial raw

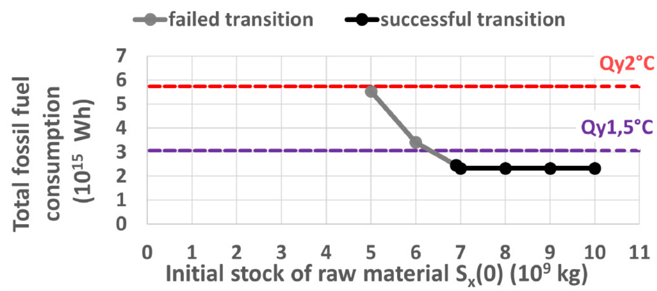


**Fig. 7.** Total fossil fuel consumption for runs with varying $S_x(0)$ with $t_f = 100$ years and without recycling. Successful transitions (black dots) and failed transitions (gray dots). Red and purple dashed lines respectively embody power generation carbon budget of France to maintain global warming below 1.5 and 2 °C.

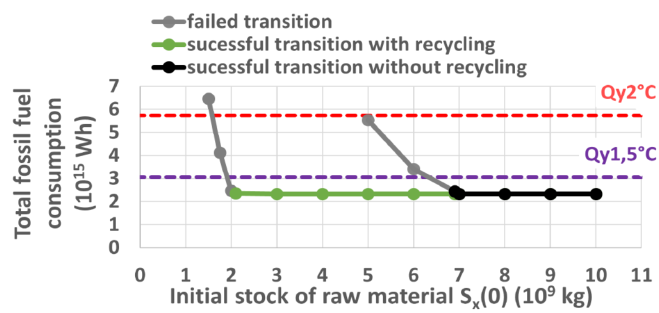


**Fig. 8.** Total fossil fuel consumption for runs with varying initial stock $S_x(0)$, without recycling (black dots) and with recycling (green dots) and with $t_f = 100$ years. Highlighting the reduction of raw material requirement to achieve successful energy transition enabled by recycling.

material stock $S_x(0)$ and the resulting total fossil fuel consumption. It shows that a minimum of $7 \times 10^9$ kg as initial raw material stock $S_x(0)$ is required to achieve and maintain a successful energy transition. Below, scarcity effect appears before 100 years.

Note that for set maximum productivity $M_{\max}$, total fossil fuel consumption is also set while energy transition is successful.

### 3.3 Scenario C: Recycling and raw material scarcity

This scenario investigate the scarcity effect in parallel with scenario B, by presenting on Figure 8 the total fossil fuel consumption for runs with varying initial stock of raw material available $S_x(0)$, considering recycling (green dots) or not (black dots) over 100 years.

On the one hand this figure shows that runs with and without recycling draw the same pattern, an asymptotic form for high initial raw material stock. However, recycling appears to achieve energy transitions when respective linear manufacturing, with mining as the only source of raw material, failed.

On the other hand, the previous remarks have to be taken with caution because results depend on the runs duration $t_f$, imposed equal to 100 years in this case. By extending it up to 400 years we can observe, on Figure 9, a slow down of the PV production after a certain time

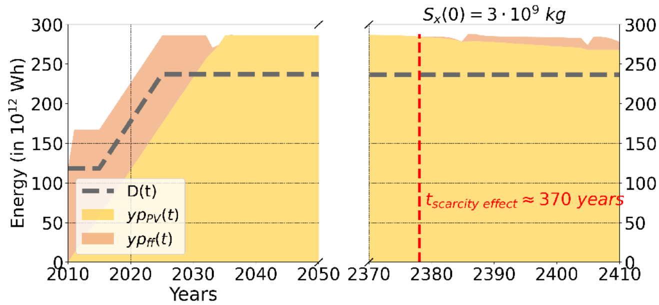


**Fig. 9.** Energy mix of France following the scenario C with $t_f = 400$ years and $S_x(0) = 3$ billion kilograms. Highlighting of delayed scarcity effect.

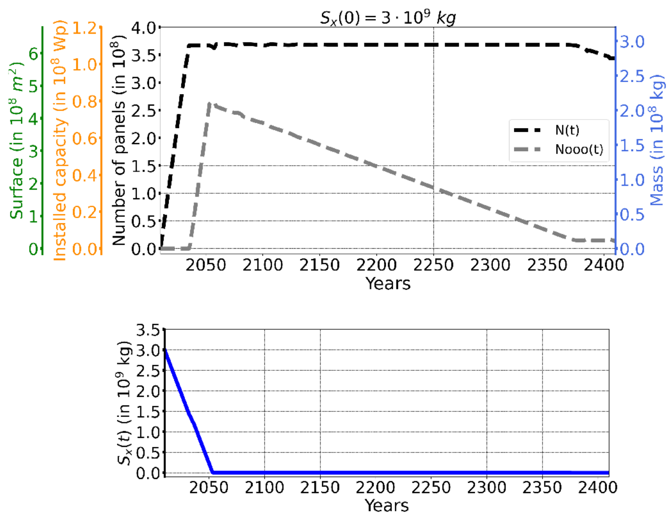


**Fig. 10.** Number of panels (up part), produced (dashed black line) and out of order panels (dashed gray line) following scenario C with $t_f = 400$ years) and $S_x(0) = 3$ billion kilograms. Raw material stock as a function of time (bottom part, blue line). Highlighting of the two steps leading to scarcity effect after 370 years.

(about 370 years), which can be considered as a delayed scarcity effect.

Figures 9 and 10 show that recycling delays scarcity effect instead of erasing it. First, we see on Figure 10 that raw material stock is depleted in 2056 (bottom part), which matches with the peak of out of order panels number (up part). From this point, these panels are recycled to produce new ones which slowly erodes its total quantity. This dissipative effect is the consequence of the hypothesis 8) which implies 5% of material loss at each cycle.

Once not enough material remains in this stock of out of order panels to maintain by recycling a constant number of functional PV panels, around 2370, then productivity slows down and consequently PV production too, leading to the return of fossil fuel consumption.

In this scenario, used panels are deemed equivalent to a second source of material, but equally limited due to dissipative effect. However its depletion rate is slower than the raw material stock's one. Hence, scarcity effect happens

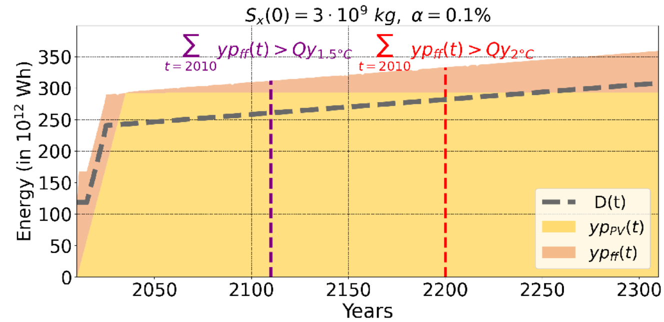


**Fig. 11.** French energy mix following the scenario D, with $t_f = 400$ years and with $S_x(0) = 3$ billion kilograms. Purple and red vertical dashed lines respectively define dates, 2110 and 2200, for which total fossil fuel consumption overtakes power generation carbon budget of France to maintain global warming below 1.5 and 2 °C.

in 2380 which is about 400 years after the no-recycling scenario B.

### 3.4 Scenario D: Influence of increasing demand $D(t)$

This latest scenario imposes a constant increasing rate $\alpha$ to the demand $D(t)$ from 2025 (permanent regime of energy demand, see Fig. 2) to the end of the simulation. Other conditions are similar to scenario C. Thus, Figure 11 presents the energy mix of france with an absolute increasing factor demand of 0.1 percent.

In these very strict conditions ($S_x(0) = 3 \times 10^9$ kg), fossil fuel consumption re-increase linearly due to limitation of productivity and $C0_2$ budgets are depleted about respectively 300 and 200 years before scarcity effect happens. Then, recycling and limited productivity over 400 years delay the climate deadlines after 2100, without solving them. Thus, the optimized solution violates constraint (a) like in scenario B and with too strict conditions.

## 4 Discussion

First of all hypothesis 1) to 6) have been chosen with the simplification that the current mature technology and existing manufacturers stay unchanged in the next century. In that respect, the resulting *Energy Returned On Energy Invested* (ERoEI) of our fictive PV panel is equal to 5.95, which is similar to PV performances on IEA report of 2011 [21].

To criticize the model, many of our hypotheses rely on the invariance of parameters over time, in other words, for all our runs technical and industrial performances, and by extension photovoltaic research results and development, have stopped to be improved since 2010 and for decades. If the model aimed at being predictive, it would raise so many uncertainties that nothing could be concluded.

Then, constraint $(h)$ energetically favors recycling as compared with mining. This might no be the case if energy storage devices would have been included in the analysis. Along the same lines, focus is made on silicon because of its active role on the photovoltaic effect but other material, like copper or aluminium, seem to be much more critical [7,22] at industrial scale. Constraint ($e$) raises a more subtle issue, applying the national energy mix to all industries (mining, manufacturing and recycling) without regard to the processes characteristics defined by an energy nature focus ; mechanical, chemical or thermal. Also, a different strategy of downscaling carbon budget from global to power generation industry changes the $Qy_{2\,^\circ\mathrm{C}}$ value and implied results [23].

In spite of these model limitations, our approach enables us to study key factors of a simplified energy transition, which are productivity limitation, panel lifetime, energy demand and planetary boundaries, expressed in our case with mineral resources and $CO_2$ emissions budget. It is dedicated to study the space and time scale of phenomena such as dissipative effect through hypothesis h) implementing that a 5% of the matter to be recycled, instead of being reuse to produce new panels, simply vanishes, leading to depletion of natural resources.

Therefore scenarios A, B and C present minimum values of raw material stock and productivity to achieve an energy transition while global warming staying below 1.5 °C over 100 years. We propose to detail the resulting *land use*, which is a stake specific to renewable energy systems versus fossil fuels: the maximum installed capacity corresponds to 618 $\mathrm{km}^2$ (see Fig. 10), ignoring space between panels, which is about 12% of the "Region Bouches-du-Rhône", a department in Southern France and about 0.11% of size of France. This result could be burden by accounting for broken panels, non-recycled matter or storage energy systems, requiring supplementary space and energy to be treated. Thus our model intends to give orders of magnitude and absolutely not to be predictive. However, improvements could be to consider progressive evolution of technological parameters such as panel lifetime, embodied energy and material or maximum productivity and introduce different strategy of end life treatments (reuse, re-manufacturing, recycling) and an energy nature focus. In addition, increasing electric demand is driven by climate change context urging to electrify transportation sector. We could account for this phenomenon with an evolving $CO_2$ budget over time, gradually absorbing the remaining budget dedicated to transport sector.

## 5 Dead end

Constraint ($p$) can be considered as a failed attempt to remove the border effect of the optimization leading to decrease consumption variable instead of keeping a steady state. The five last years of some runs could be deleted to this end. Moreover, material depletion conflicts with the constraint ($a$), leading to what we called a scarcity effect and thus to a partial optimized solution.

## 6 Conclusion

This toy-model illustrates the interactions between raw material stock, industrial productivity and recycling, energy demand and climate objective. Consequently, we could conclude that energy transition, defined as reaching an energy mix fully sourced by renewable energy, can be achieved in some decades without performance improvements of existing photovoltaïc panels based on runs over 100 years.

One aspect that ought, however, to receive our full attention is that results also pinpoint to the limitation that the productivity affects the transition duration, at the same level as panel lifetime. Indeed, on the one hand, we demonstrate that time horizon of a run influences the upholding of the *steady state* after transition. On the other hand, scarcity phenomenon despite massive recycling, increasing energy demand and disregarded issues, including storage systems contributing to stability network, or environmental and land-use constraints, threat the sustainability of our modelled energy production industry.

Sustaining this steady state within planetary boundaries and on the long term implies significant improvements of industrial practices, such as increasing the maximum productivity and intensifying recycling, but also requires improvements of technical performances, especially the lifetime of panels but also for instance power density and ecological footprint, which depends mainly on the renewable energy research.

Lastly, this extrapolating model can also be used to compare technologies, for instance monocristalline and bi-facial panel technologies, or other renewable energy systems with a focus on a common material and in the same context to investigate respective land-uses and energy transitions resulting from their performances.

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